Int. J. Heat Mass Transfer. Vol. 23, pp. 853–860
Pergamon Press Ltd. 1980. Printed in Great Britain

# RIGOROUS AND SIMPLIFIED APPROACH TO THE RADIATIVE TRANSFER IN AN ABSORBING AND ANISOTROPICALLY SCATTERING SLAB WITH A REFLECTING BOUNDARY*

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(*Received* 25 *July* 1979)

**Abstract**—Radiative transfer has been considered within a plane participating slab assuming diffuse emission at one boundary and diffuse reflection at the other boundary. The medium is assumed to be a zero temperature one so that emission is neglected and only absorption and scattering (according to a linearly anisotropic law) are assumed within the slab.

A rigorous solution is developed following a constructive procedure based on projectional methods: the resulting formulae have been numerically processed to obtain the distribution of the physically relevant variables for some significant situations.

The results from the rigorous approach are then used as reference values to test the reliability of the results from some of the simplified approaches used in the literature ($P_1$ approximation and kernel substitution).

## NOMENCLATURE

- $a$, optical half-thickness;
- $c$, albedo;
- $E_n$, $n$th exponential integral;
- $I$, angular radiation intensity;
- $I_0$, total radiation intensity;
- $q = I_1$, net radiative flux;
- $q^+, q^-$, forward, backward radiative flux.

Greek symbols

- $\alpha$, emitted power;
- $\mu$, cosine of the angle between the direction of the radiation intensity and the positive $\tau$ axis;
- $\bar{\mu}_0$, weighted average of the cosine of the scattering angle;
- $\rho$, diffuse reflectivity of the boundary $\tau = a$;
- $\tau$, optical coordinate.

## 1. INTRODUCTION

IN SOME recent papers rigorous semi-analytical approaches to the solution of the radiant energy balance equation in a plane participating slab have been proposed, when radiation can be emitted and reflected by the bounding surfaces [1, 2]. On the other hand many practical problems of physical interest in radiative transfer are usually investigated through pure numerical schemes, or by resorting to very simple approximate techniques [3–5], such as kernel substitution, and $P_1$ approximation. In any case, many authors confine themselves to the evaluation of integral quantities, like hemispherical reflectivity and transmissivity, and accurate results for the angular radiation intensity are seldom available. This paper deals with the problem of a low temperature participating medium, which can scatter anisotropically and absorb, subject to a diffuse radiation emitted by one of its boundaries, while the other is assumed to reflect in a diffuse way. This physically-meaningful problem has been already studied, at least in some particular cases, and partial results have been given through either rigorous (Case's method), or approximate, or fully numerical procedures. The present solution is constructed following the exact theory presented in [2] and [6], which seems to be the most straightforward and flexible among the rigorous ones, and a complete solution is easily obtained for the directional fluxes and all main quantities relevant to radiative transfer. Comparison is made with previous existing results; $P_1$ approximation and kernel-substitution techniques are also worked out to cover the situations examined. Since the present results are very accurate and reliable they should be used as reference results to test the validity and the accuracy of the numerical and simplified approaches given in the literature.

## 2. THE RIGOROUS APPROACH

### 2.1. *Theory*

According to the theory presented in [6], the

*Work supported by C.N.R., Roma (Grant 78.02453.07/115.9364).

angular radiation intensity can be expressed, for the physical problem described in the Introduction and in the case of linearly anisotropic scattering, in terms of its first two Legendre moments as

$$I(\tau,\mu)=\frac{c}{4\pi}\frac{1}{\mu}\int_{-a}^{\tau}e^{-(\tau-\tau')/\mu}I_0(\tau')d\tau'+\frac{c}{4\pi}3\bar{\mu}_0\int_{-a}^{\tau}e^{-(\tau-\tau')/\mu}I_1(\tau')d\tau'+\frac{\alpha}{\pi}e^{-(a+\tau)/\mu},\tag{1a}$$

$$\begin{aligned}I(\tau,-\mu)=&\frac{c}{4\pi}\frac{1}{\mu}\int_{\tau}^{a}e^{(\tau-\tau')/\mu}I_0(\tau')d\tau'+2\rho\frac{c}{4\pi}e^{-(a-\tau)/\mu}\int_{-a}^{a}E_2(a-\tau')I_0(\tau')d\tau'\\&-\frac{c}{4\pi}3\bar{\mu}_0\int_{\tau}^{a}e^{(\tau-\tau')/\mu}I_1(\tau')d\tau'+2\rho\frac{c}{4\pi}3\bar{\mu}_0e^{-(a-\tau)/\mu}\\&\times\int_{-a}^{a}E_3(a-\tau')I_1(\tau')d\tau'+2\rho\frac{\alpha}{\pi}E_3(2a)e^{-(a-\tau)/\mu},\end{aligned}\tag{1b}$$

where $-a\leqslant\tau\leqslant a, 0<\mu\leqslant 1$, $\bar{\mu}_0$ is the average cosine of the scattering angle, $I_0$ is the total incident radiation, $I_1=q$ the net radiative heat flux, and $E_n$ denotes the general exponential integral function [7]. Boundary conditions are already incorporated in equations (1), where $\alpha$ is the total power emitted by the boundary $\tau=-a$, and $\rho$ the diffuse reflectivity of the boundary $\tau=a$. The moments $I_0$ and $I_1$ are in turn solutions to the system of coupled linear integral equations

$$I_0(\tau)=\frac{1}{2}c\int_{-a}^{a}H_{00}(\tau,\tau')I_0(\tau')d\tau'+\frac{3}{2}\bar{\mu}_0c\int_{-a}^{a}H_{01}(\tau,\tau')I_1(\tau')d\tau'+F_0(\tau),\tag{2a}$$

$$I_1(\tau)=\frac{1}{2}c\int_{-a}^{a}H_{10}(\tau,\tau')I_0(\tau')d\tau'+\frac{3}{2}\bar{\mu}_0c\int_{-a}^{a}H_{11}(\tau,\tau')I_1(\tau')d\tau'+F_1(\tau),\tag{2b}$$

whose kernels are the same as in [6], whereas the known terms take the form

$$F_i(\tau)=2\alpha E_{2+i}(a+\tau)+(-1)^i4\rho\alpha E_3(2a)E_{2+i}(a-\tau),\quad i=0,1.\tag{3}$$

The system is then solved by projection, by resorting to the Legendre polynomials $P_n(\tau/a)$ as coordinate functions. The result is

$$\begin{aligned}I_i(\tau)=&\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^0\left\{\frac{1}{2}cU_n^i(\tau)+(-1)^i\rho cD_n^0E_{2+i}(a-\tau)\right\}\\&+\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^1\left\{\frac{3}{2}\bar{\mu}_0cU_n^{1+i}(\tau)+(-1)^i3\bar{\mu}_0\rho cD_n^1E_{2+i}(a-\tau)\right\}+F_i(\tau),\\&i=0,1\end{aligned}\tag{4}$$

and then

$$I(\tau,\mu)=\frac{c}{4\pi}\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^0W_n(\tau,\mu)+\frac{c}{4\pi}3\bar{\mu}_0\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\times\eta_n^1\mu W_n(\tau,\mu)+\frac{\alpha}{\pi}e^{-(a+\tau)/\mu},\tag{5a}$$

$$\begin{aligned}I(\tau,-\mu)=&\frac{c}{4\pi}\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^0\{(-1)^nW_n(-\tau,\mu)+2\rho D_n^0e^{-(a-\tau)/\mu}\}\\&-\frac{c}{4\pi}3\bar{\mu}_0\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^1\{(-1)^n\mu W_n(-\tau,\mu)-2\rho D_n^1e^{-(a-\tau)/\mu}\}+2\rho\frac{\alpha}{\pi}E_3(2a)e^{-(a-\tau)/\mu}\end{aligned}\tag{5b}$$

for the angular radiation intensity. The partial radiative fluxes $q^+(\tau)$ and $q^-(\tau)$, with $q=q^+-q^-$, can be also explicitly evaluated as

$$q^+(\tau)=\frac{c}{2}\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^0G_n^0(\tau)+\frac{c}{2}3\bar{\mu}_0\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\times\eta_n^1G_n^1(\tau)+2\alpha E_3(a+\tau),\tag{6a}$$

$$\begin{aligned}q^-(\tau)=&\frac{c}{2}\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^0\times\{(-1)^nG_n^0(-\tau)+2\rho D_n^0E_3(a-\tau)\}\\&-\frac{c}{2}3\bar{\mu}_0\sum_{n=0}^{N}\left(\frac{2n+1}{2a}\right)^{1/2}\eta_n^1\{(-1)^nG_n^1(-\tau)-2\rho D_n^1E_3(a-\tau)\}+4\rho\alpha E_3(2a)E_3(a-\tau).\end{aligned}\tag{6b}$$

The functions $U_n^i(\tau)$, $W_n(\tau,\mu)$, $G_n^i(\tau)$ and the numbers $D_m^i$ are integrals involving exponential integrals and Legendre polynomials; they can all be written in closed analytical form by means of elementary and special functions, as shown in [6].

At this point only the evaluation of the unknown numbers $\eta_n^0$ and $\eta_n^1$, which are needed as expansion coefficients, must be performed. They follow easily from the solution of the linear algebraic system

$$\eta_m^i=F_m^i+\sum_{j=0}^{1}\sum_{n=0}^{N}\frac{[(2m+1)(2n+1)]^{1/2}}{2a}G_{mn}^{ij}\eta_n^j,\quad m=0,1,\ldots,N,\quad i=0,1.\tag{7}$$

where the matrix elements $G^{ij}_{mn}$ are given once more in [6], and the known terms are now

$$F^i_m = \left(\frac{2m+1}{2a}\right)^{1/2} 2\alpha D^i_m[(-1)^m + (-1)^i 2\rho E_3(2a)]. \tag{8}$$

### 2.2. *Results*

In order to investigate the effects of forwardness of the scattering and of the reflectivity of the boundary, the extreme values of $\bar{\mu}_0$ [i.e. $\bar{\mu}_0 = 1/3$ (forward scattering), $-1/3$ (backward scattering)] and $\rho$ (i.e. $\rho = 0, 1$) have been considered for any of the optical thickness examined. Investigation has been limited to situations where scattering phenomena are relevant ($c = 0.9$) and furthermore, the case of isotropic scattering ($\bar{\mu}_0 = 0$) has always been considered as a reference case, even if, for the sake of clearness, the results for this case are often omitted in the following figures since they can be located easily between those for the two extreme values of $\bar{\mu}_0$.

All the quantities relevant to radiative transfer ($I_0, q$) are defined as moments of different order of the angular intensity $I(\tau, \mu)$ and therefore attention should be given to the distributions of such a variable.

Figure 1 gives the distribution of $I(\tau, \mu)$ at different positions within the slab for $\bar{\mu}_0 = 1/3$ and $\bar{\mu}_0 = -1/3$ in both the cases of a transparent ($\rho = 0$) and totally reflecting ($\rho = 1$) boundary at $\tau = a$. Results are given only for $a = 1$, but the same effects will occur for any value of $a$, even if their relevance depends on $a$, with special regard to the influence of $\rho$.

Since the probability of a photon to proceeding further within the slab increases when the forwardness increases, in the transparent boundary case, $I(\tau, \mu)$ is, for any $\tau$, higher for $\bar{\mu}_0 = 1/3$ than for $\bar{\mu}_0 = -1/3$ when $\mu > 0$, while the opposite occurs when $\mu < 0$. When a totally reflecting boundary is considered this trend still occurs in points far away from the reflecting boundary, while close to it, in a region whose width depends on $a$, the $I(\tau, \mu)$ distribution is, for any $\mu$, higher in the forward scattering case than in the backward one since more photons are available for reflection when $\bar{\mu}_0 = 1/3$.

The distributions occurring for $q^+$, $q^-$ and $q$ are

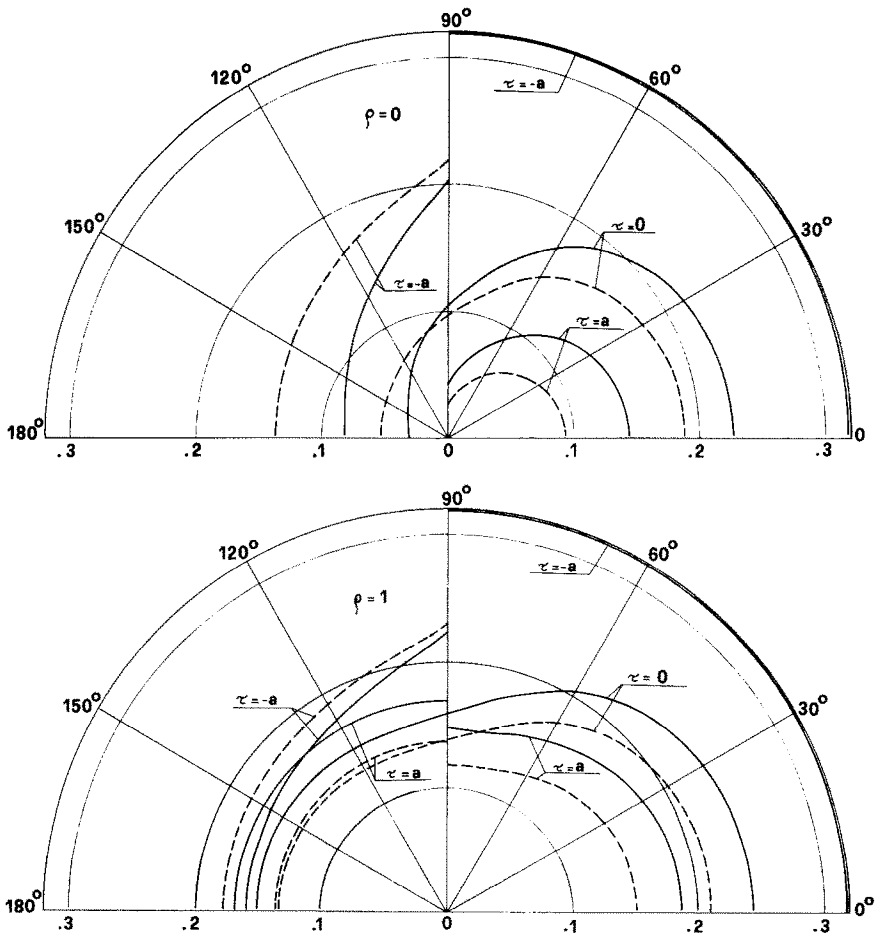


Fig 1

FIG. 1. Angular distribution for the radiation intensity $I(\tau, \mu)$ for $a=1.0$, $c=0.9$. (—— $\bar{\mu}_0=1/3$; – – $\bar{\mu}_0=1/3$).

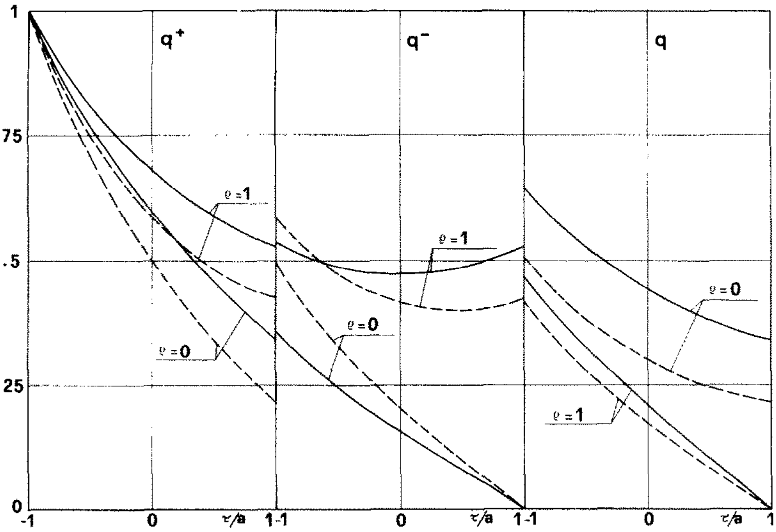


FIG. 2. Forwards ($q^+(\tau)$), backwards ($q^-(\tau)$) and net ($q(\tau)$) radiative flux for $a = 1.0$, $c = 0.9$. (—— $\bar{\mu}_0 = 1/3$; – – – $\bar{\mu}_0 = -1/3$).

given in Fig. 2 and the resulting trends can be explained straightforwardly on the basis of the results given in Fig. 1.

Whatever the value of $\rho$, the $q$ curve for the backward scattering is always lower than that for the forward case, the difference between the two curves decreasing when $\rho$ is increased.

The influence of the optical thickness on the $I_0$ distribution can be grasped from Fig. 3, where the distributions of such a variable are given for three different values of $a$, significant of situations where the optically thin and thick limits are approached as well as of an intermediate situation.

The crossing occurring between the curves at different values of $\bar{\mu}_0$ for any $\rho$ and $a$ can be explained once more on the basis of the arguments used to justify the results of Fig. 1: the curve for $\bar{\mu}_0 = 1/3$ is lower than that for $\bar{\mu}_0 = -1/3$ close to the boundary $\tau = -a$, while the opposite occurs close to boundary $\tau = a$, since in these two regions of the slab the photons available for absorption in the forward-scattering case are respectively less, and more than in the backward case (it must be recalled that $I_0$ is proportional, through $(1 - c)$, to the local volumetric rate of energy absorption). For $\rho = 1$ the curves are strongly increased and flattened for low values of $a$, while they are

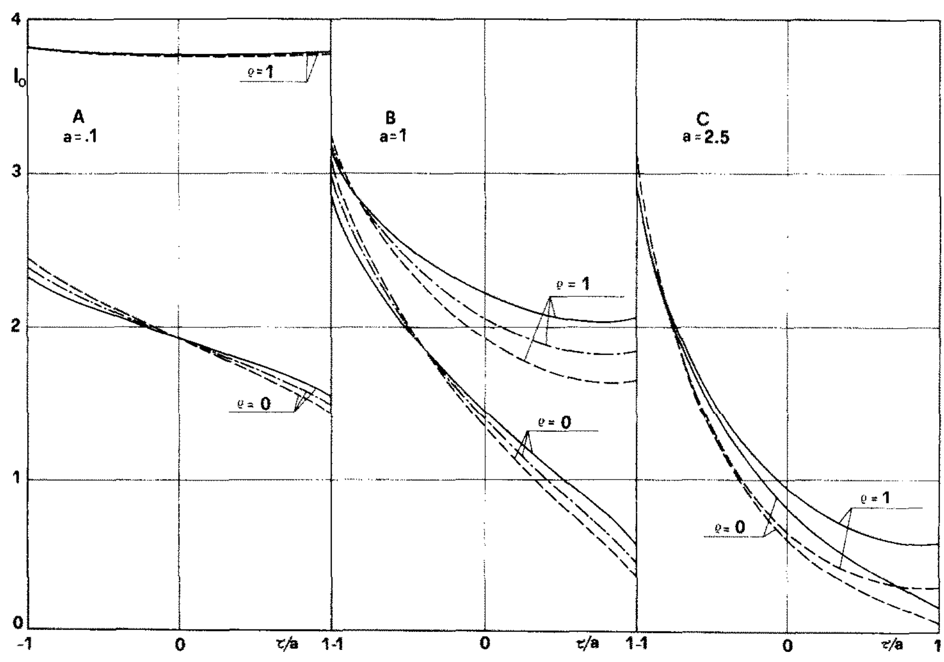


FIG. 3. The total radiation intensity $I_0(\tau)$ as a function of $\tau/a$ for $c = 0.9$. —— $\bar{\mu}_0 = 1/3$; — · — $\bar{\mu}_0 = 0$; — — $\bar{\mu}_0 = -1/3$.

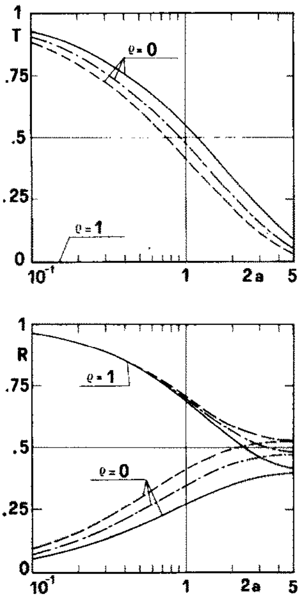


FIG. 4. The hemispherical transmissivity $T$ and the hemispherical reflectivity $R$ of the slab as a function of the optical thickness $2a$. —— $\bar{\mu}_0 = 1/3$; —·— $\bar{\mu}_0 = 0$; — — $\bar{\mu}_0 = -1/3$.

noticeably raised only close to the reflecting wall for intermediate-high values of $a$. Finally, it is worth considering the hemispherical transmissivity $T$ and hemispherical reflectivity $R$ of the slab defined as

$$T = \frac{2\pi \int_{-1}^{1} I(a,\mu)\,\mu\,\mathrm{d}\mu}{2\pi \int_{0}^{1} I(-a,\mu)\,\mu\,\mathrm{d}\mu} = \frac{q(a)}{\alpha}, \tag{9a}$$

$$R = \frac{2\pi \int_{0}^{1} I(-a,-\mu)\,\mu\,\mathrm{d}\mu}{2\pi \int_{0}^{1} I(-a,\mu)\,\mu\,\mathrm{d}\mu} = \frac{q^{-}(-a)}{\alpha} \tag{9b}$$

respectively. These two quantities give the fraction of the energy emitted at $\tau = -a$, which is lost through the boundary $\tau = a$ and $\tau = -a$ respectively. The results are given in Fig. 4: for $\rho = 0$, $T$ is, for any $a$, higher for $\bar{\mu}_0 = 1/3$ than for $\bar{\mu}_0 = -1/3$ and is, of course, decreasing when the optical thickness is increased, while $R$ is higher in the backward-scattering case and increases with $a$.

For $\rho = 1$, $T$ is zero in any case, while $R$ is strongly increased in the intermediate range of $a$ and in the optical-thin limit, where no dependence on the scattering mode results since, owing to the negligible absorption phenomena, the reflecting wall acts as an emitting wall equivalent to the one at $\tau = -a$. For high values of $a$, on the contrary, the presence of the reflecting wall does not affect the value of $R$ since, owing to the increased absorption within the participating medium, the reflected photons do not succeed in reaching the boundary $\tau = -a$. It must be noted that, for the isotropic-scattering case, the computed values of $T$ and $R$ completely reproduce those given in [3].

## 3. SIMPLIFIED APPROACHES AND COMPARISON OF THE RESULTS WITH THE RIGOROUS SOLUTIONS

Many approximate techniques are currently used to solve radiative-transfer problems: some of them are reviewed, for instance in [8]. The use of any of these approximations is very appealing since it implies a drastic simplification of the mathematics of the problem: the reliability of the solutions obtained following such approaches would nevertheless be checked against a solution based on a rigorous approach. The solutions obtained following the approach outlined in Section 2 have been used therefore as a reference case to test the solutions obtained following two approximate techniques, namely the kernel substitution and $P_1$ approximation.

The kernel-substitution approach has been used by Dayan and Tien who studied the case of a slab with emitting, transparent boundaries, considering both the cases of pure radiative transfer [5] and of combined radiative and conductive transport [4]. The approximate solutions were obtained by replacing the exponential integrals within the two starting integral equations with exponential functions, since it was assumed that

$$E_1(\tau) \simeq 2\exp(2\tau). \tag{10}$$

Results were then checked against those obtained from a purely numerical solution of the two starting integral equations.

The problem has therefore been reworked to include the case of a reflecting boundary, and, following the same procedure as in [5], the problem has been reduced to the solution of two simple ODE, as in [5], i.e. in our notation:

$$\frac{\mathrm{d}^2 I_0}{\mathrm{d}\tau^2} = b^2 I_0, \qquad \frac{\mathrm{d}^2 q}{\mathrm{d}\tau^2} = b^2 q \tag{11}$$

with

$$b^2 = (1-c)(4-3c\bar{\mu}_0).$$

In this case the solution of these equations can be given as

$$I_0(\tau) = \frac{b}{1-c}\{A_1 \exp[-b(a+\tau)] - A_2 \exp[-b(a-\tau)]\},$$
$$q(\tau) = A_1 \exp[-b(a+\tau)] + A_2 \exp[-b(a-\tau)] \tag{12}$$

where

$$A_1 = \frac{4\alpha - A_2 X_2}{X_1},$$

$$A_2 = \frac{-4\alpha[X_2 + \rho X_1 \exp(-2ba)]}{X_1^2 - X_2^2 + 2\rho X_1 X_2 \sinh(2ba)} \tag{13}$$

with

$$X_1 = \frac{1}{2-b}\left[3c\bar{\mu}_0 + (4 - 3c\bar{\mu}_0)\frac{2c}{b}\right],$$
$$X_2 = \frac{\exp(-2ba)}{2+b}\left[3c\bar{\mu}_0 - (4 - 3c\bar{\mu}_0)\frac{2c}{b}\right]. \quad (14)$$

The $P_1$ approximation has been used by Lii and Ozisik [3] for evaluating the hemispherical transmissivity and reflectivity of a slab with a reflecting boundary under the assumption of isotropic scattering and results were tested against those from Case's method.

When linearly anisotropic scattering is considered, the $P_1$ approximation requires the simple

$$\frac{d^2 I_0}{d\tau^2} = 3(1 - \bar{\mu}_0 c)(1 - c) I_0 \quad (15)$$

to be solved under the following boundary conditions, expressed through Marshak's approximation,

$$I_0(-a) - \frac{2}{3}\frac{1}{1-\bar{\mu}_0 c}\left.\frac{dI_0}{d\tau}\right|_{\tau=-a} = 4\alpha,$$
$$(1-\rho)I_0(a) + \frac{2}{3}\frac{1+\rho}{1-\bar{\mu}_0 c}\left.\frac{dI_0}{d\tau}\right|_{\tau=a} = 0. \quad (16)$$

The solution can then be given as

$$I_0(\tau) = A_1 \exp(b\tau) + A_2 \exp(-b\tau) \quad (17)$$

where

$$b^2 = 3(1 - \bar{\mu}_0 c)(1 - c) \quad (18)$$

and

$$A_1 = \frac{4\alpha X_4}{X_1 X_4 - X_2 X_3}, \quad A_2 = -\frac{4\alpha X_3}{X_1 X_4 - X_2 X_3} \quad (19)$$

with

$$X_1 = \exp(-ba)\left(1 - \frac{2}{3}\frac{b}{1-\bar{\mu}_0 c}\right),$$
$$X_2 = \exp(ba)\left(1 + \frac{2}{3}\frac{b}{1-\bar{\mu}_0 c}\right),$$
$$X_3 = \exp(ba)\left(\frac{1-\rho}{1+\rho} + \frac{2}{3}\frac{b}{1-\bar{\mu}_0 c}\right),$$
$$X_4 = \exp(-ba)\left(\frac{1-\rho}{1+\rho} - \frac{2}{3}\frac{b}{1-\bar{\mu}_0 c}\right). \quad (20)$$

Once the distribution of $I_0(\tau)$ has been evaluated, $q(\tau)$ can be obtained as

$$q(\tau) = -\frac{b}{3(1-\bar{\mu}_0 c)}(A e^{b\tau} - B e^{-b\tau}). \quad (21)$$

The distribution of $I_0(\tau)$ and $q(\tau)$ have been then evaluated for $c = 0.9$, $\rho = 1$ and different optical thicknesses, assuming both forward and backward scattering.

Results are given in Figs. 5 and 6 where the results from the rigorous approach are also given. Of course approximate results are acceptable for low optical thickness since they essentially depend on the walls' properties while their accuracy is otherwise quite poor for intermediate–high values of $a$, where a significant

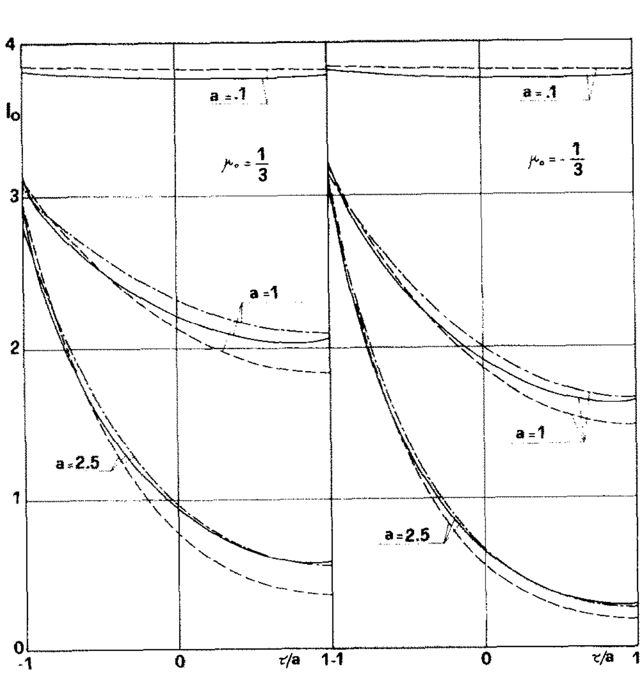


FIG. 5. Rigorous and simplified approaches: the total radiation intensity $I_0(\tau)$ as a function of $\tau/a$ for $c = 0.9$ and $\rho = 1$. —— rigorous; —·— $P_1$ approximation; – – – kernel substitution.

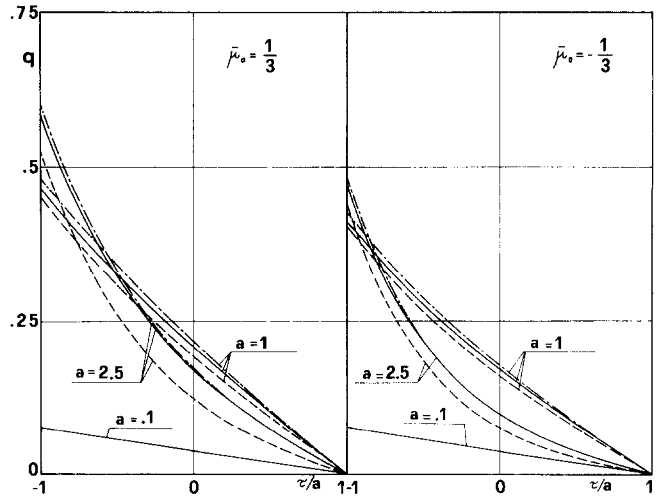


FIG. 6. Rigorous and simplified approaches: the net radiative flux $q(\tau)$ as a function of $\tau/a$ for $c = 0.9$ and $\rho = 1$. —— rigorous; —·— $P_1$ approximation; —— kernel substitution.

role is played by the participating medium. As could be expected the $P_1$-approximation becomes more satisfactory the higher $a$.

Recalling the different mathematical sophistication of the considered approaches, any decision about relying on or rejecting an approximate solution is left to the user's judgement, depending on his particular needs.

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## APPROCHES RIGOUREUSE ET SIMPLIFIEE DU TRANSFERT PAR RAYONNEMENT DANS UNE PLAQUE ABSORBANTE ET ANISOTROPIQUEMENT DIFFUSANTE, AVEC FRONTIERE REFLECHISSANTE


**Résumé**—On considère le transfert par rayonnement dans une plaque plane, avec émission diffuse à une frontière et une réflexion diffuse à l'autre. Le milieu est supposé à température nulle de telle sorte que l'émission est négligée et on ne considère que l'absorption et la diffusion. Une solution rigoureuse est développée en suivant une procédure basée sur des méthodes projectionnelles: les formules résultantes ont été obtenues numériquement pour obtenir la distribution des variables physiques intéressantes pour quelques cas significatifs. Les résultats de l'approche rigoureuse sont ensuite utilisés en référence pour tester des approches simplifiées utilisées dans la bibliographie (approximation $P_1$ et substitution du noyau).


## STRENGE UND VEREINFACHTE LÖSUNG DES STRAHLUNGSWÄRMEÜBERGANGS IN EINER ABSORBIERENDEN UND ANISOTROP STREUENDEN PLATTE MIT EINER REFLEKTIERENDEN BEGRENZUNG


**Zusammenfassung**—In einer ebenen Platte, bei angenommener diffuser Emission an der einen Begrenzung und diffuser Reflexion an der anderen, wurde der Strahlungswärmeaustausch behandelt. Das Medium soll die Temperatur Null besitzen, so daß Emission vernachlässigt werden kann und nur Absorption und Streuung (gemäß einem linearen anisotropen Gesetz) innerhalb der Platte angenommen werden. Es wird eine strenge Lösung entwickelt, die sich an ein Verfahren anlehnt, welches auf Projektionsmethoden beruht: Die daraus gewonnenen Formeln sind numerisch ausgewertet worden, um die Verteilung der physikalisch relevanten Größen für einige wichtige Fälle zu erhalten.

Die Ergebnisse der strengen Lösung werden als Vergleichswerte benutzt, um die Zuverlässigkeit der Ergebnisse einiger vereinfachter Lösungen zu testen, die in der Literatur benutzt werden ($P_1$-Approximation und Kernel-Substitution).

# СТРОГИЙ И УПРОЩЕННЫЕ ПОДХОДЫ К ИССЛЕДОВАНИЮ ПЕРЕНОСА ИЗЛУЧЕНИЯ В ПОГЛОЩАЮЩЕЙ И АНИЗОТРОПНО РАССЕИВАЮЩЕЙ ПЛИТЕ С ОТРАЖАЮЩЕЙ ПОВЕРХНОСТЬЮ

**Аннотация** — Рассмотрен перенос излучения внутри плоской проницаемой плиты в предположении диффузной эмиссии на одной поверхности и диффузного отражения на другой. Предполагается, что среда находится при нулевой температуре, так что можно пренебречь излучением, а также, что имеют место только абсорбция и рассеяние (в соответствии с линейно анизотропным законом). С помощью проекционных методов получено строгое решение. В результате численной обработки расчетных формул определены физические переменные для ряда практически важных случаев.

Результаты, полученные на основании строгого подхода, использовались затем для проверки надежности некоторых упрощенных методов, полученных другими авторами ($P_1$-аппроксимации и подстановки Кернеля).